Methods for analysis and control of dynamical systems
Lecture 5: Delay and saturation issues

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5th February 2015
Outline

Introduction

Integrator anti-windup
  Problem overview
  Anti-windup schemes

Time-delay
  Introduction
  About stability
  Simple PID controller

References
References

Some interesting books:

Some examples:

1. Maximal torque delivered by a motor
2. Valves is saturating when it is fully opened or closed
3. Maximal angles of the rudder plane
4. Maximal gain of the electronic amplifiers
5. Maximal power of an energy unit
6. Maximal force of a spring or damper
7. Maximal flow in ducts
8. ....
Associated problem: poor performances

\( u(t) \) belongs to \([u_{\text{min}}, u_{\text{max}}]\).

When a given reference is too large, the controller will generate a large control input \( u(t) \), greater than \( u_{\text{max}} \). Due to the integrator, \( u(t) \) will continue growing until the plant output comes closer to the reference, which will take more time.

Problem: when the actuator saturates the system is in OPEN-LOOP conditions.
Let us consider the simple integrator:

\[ G(s) = \frac{1}{s} \]

controlled by a PI controller:

\[ K(s) = K_p + \frac{K_i}{s} \]

where \( K_p = 2 \), \( K_i = 4 \).
Assume that the control action is limited by \( u_{max} = 1 \).
Use of Matlab for illustration
Matlab example: suspension control

Figure: The quarter vehicle model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_s$</td>
<td>315 kg</td>
<td>sprung mass</td>
</tr>
<tr>
<td>$m_u$</td>
<td>37.5 kg</td>
<td>unsprung mass</td>
</tr>
<tr>
<td>$k$</td>
<td>29500 N/m</td>
<td>suspension linearized stiffness</td>
</tr>
<tr>
<td>$c$</td>
<td>1500 N/m/s</td>
<td>suspension linearized damping</td>
</tr>
<tr>
<td>$k_p$</td>
<td>210000 N/m</td>
<td>tire stiffness</td>
</tr>
<tr>
<td>$Z_{defSusp_{max}}$</td>
<td>0.1 m</td>
<td>maximum deflection of the suspension</td>
</tr>
</tbody>
</table>

Table: Parameters model
Suspension control

"Skyhook" control algorithm: consists of designing an active suspension control so that the chassis is "linked" to the sky in order to reduce vertical oscillations of the chassis and of the axle independently of each other.

Figure: Skyhook suspension
Suspension control

The control law:

\[ u = -c_{sky} \ddot{z}_s + \alpha c_{sky} \dot{z}_u \tag{1} \]

Where \( c_{sky} > 0 \) and \( \alpha \in [0; 1] \) are design parameters that directly influence the performances.

If measurements = suspension vertical accelerations.

\[ u = -\frac{c_{sky}}{s} \dddot{z}_s + \frac{\alpha c_{sky}}{s} \ddot{z}_u \tag{2} \]

However the suspension force is limited by \( F \leq 1000N \).

Effect in simulations....
Towards anti-windup

Objective: turn-off the integrator when the actuator saturates.
Purpose: provide a local feedback to make the controller stable alone when the main loop is opened by signal saturation.
Integrator AW schemes
Integrator AW schemes
Integrator AW schemes
Integrator AW schemes
Back to the examples

- Integrator: use of a AW gain $K_a = 10$ solves the problem
- Suspension: the problem on the action is solved but it remains a steady state error.... then it is necessary to have a controller which is able to reject input disturbances.
Niculescu [2001]
Time-delays appears in many systems: industry, biology, robotics, economy ....
It allows to describe propagation and transport phenomena or population dynamics (reproduction, development or extinction).
In communication, data transmission is always accompanied by a non-zero time interval between the initiation- and the delivery-time of a message or signal.
Examples
Application to the teleoperation of a vehicle (steer-by-wire)
Many systems can be represented as:

\[ G(s) = e^{-sh}H(s) \]

where \( H \) is a usual transfer function and \( h \) the input-output delay.
Effect of time-delay

- Poor performances
- Instability
- Difficulties in control design
Control of dynamical systems

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References

References

Closed-loop system

Note \( L(s) = G(s)K(s) \) where \( K \) is the controller. Define:

\[
L(s) = e^{-sh} \frac{N(s)}{D(s)}
\]

The Nyquist criterion remains valid for a pure time-delay system, since \( e^{-sh} \) does not introduce any additional poles and zeros

Closed loop system:

\[
T(s) = \frac{N(s)e^{-sh}}{D(s) + N(s)e^{-sh}}
\]

The characteristic ”polynomial” is:

\[
p(s, h) = D(s) + N(s)e^{-sh}
\]

Definition

\( p(s, h) \) is said to be stable if

\[
p(s, h) \neq 0, \quad \forall s \in \mathbb{C}_+
\]
If the system is stable for $h = 0$, then the delay margin is:

$$
\bar{h} = \min\{ h \mid p(j\omega, e^{-jh\omega}) = 0, \text{ for some } \omega \in \mathbb{R} \}
$$

Frequency-sweeping test: find the critical delay value at which the characteristic roots intersect the stability boundary, i.e. the imaginary axis, thus rendering the system unstable.
Tsypkin method

Frequency-sweeping

Proposition

*If* $D(s)$ *is a stable polynomial the the closed-loop system*:

$$T(s) = \frac{N(s)e^{-sh}}{D(s) + N(s)e^{-sh}}$$

*is stable for any value of the delay* $h$ *if and only if*

$$|D(j\omega)| > |N(j\omega)|, \quad \forall \omega \in \mathbb{R}$$
System:

\[ G(s) = e^{-s \cdot h} \frac{K}{\tau s + 1} \]

assumption: open loop stable system
Ziegler - Nichols

Steps:

1. Close the loop with a small gain controller
2. Increase the gain until the loop (often the control signal) starts oscillating. $K_u$ is the ultimate gain and $T_u$ the ultimate period.
3. Set the control parameters following:

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>$0.5K_u$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td></td>
<td>$0.45K_u$</td>
<td>$T_u/1.2$</td>
</tr>
<tr>
<td>PID</td>
<td>$0.6K_u$</td>
<td>$0.5T_u$</td>
<td>$0.125T_u$</td>
</tr>
</tbody>
</table>
Analytical tuning

Determine the gain margin $A_m$, the phase crossover frequency $\omega_p$, phase margin $\phi_m$.

Using the approximation of the $\arctan$ function, the controller parameters can be determined by:

$$K_p = \frac{\tau \omega_p}{A_m K}$$

$$T_i = (2\omega_p - \frac{4\omega_p^2 h}{\pi} + \frac{1}{\tau})^{-1}$$

$$\omega_p = \frac{A_m \phi_m + 0.5\pi A_m (A_m - 1)}{(A_m^2 - 1) h}$$
Control difficulties

Consider a PI controller \( C(s) = K_p(1 + \frac{1}{T_i s}) \). The characteristic equation becomes

\[
T_i s(\tau s + 1) + KK_p(T_i s + 1)e^{-sh} = 0
\]

which is not simple to solve ....

A usual method consists in cancelling the lag, i.e to choose \( T_i = \tau \) which corresponds to the CL system:

\[
T(s) = \frac{KK_p e^{-sh}}{\tau s + KK_p e^{-sh}}
\]

The system is stable only if

\[
0 \leq KK_p \leq \frac{\pi \tau}{2 h}
\]

The longer the delay the smaller the maximum allowable gain.
Smith predictors

**Objective**: try to reduce the ”presence” of the delay in the closed-loop system, as if the delay were shifted outside the feedback loop.

Consider

\[ G(s) = e^{-sh}H(s) \]

Denote \( C_0(s) \) the controller designed using \( H(s) \) only. Define \( Z(s) = H(s) - e^{-sh}H(s) \) the predictor and the controller:

\[ C(s) = \frac{C_0(s)}{1 + C_0(s)Z(s)} \]

The closed-loop system is then:

\[ T(s) = \frac{C_0(s)H(s)}{1 + C_0(s)H(s)} e^{-sh} \]
Smith predictors (cont.)

\[ \begin{align*}
    r & \rightarrow +/-. \rightarrow +/-. \rightarrow C_0(s) \rightarrow u \rightarrow H(s)e^{-sb} \rightarrow y \\
    & \text{(Diagram continues here)}
\end{align*} \]
Smith predictors

Mtalab example